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One loop gluon form factor and freezing of α_s in the Gribov-Zwanziger QCD Lagrangian

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Abstract: We use the Gribov-Zwanziger Lagrangian in QCD to evaluate the one loop correction to the gluon propagator as a function of the Gribov volume and verify that the propagator vanishes in the infrared limit. Using the corresponding correction to the Faddeev-Popov ghost propagator we construct the renormalization group invariant coupling constant, $\alpha_S^{\text{eff}}(p^2)$, from the gluon and ghost form factors and verify, using the Gribov mass gap condition, that it freezes out at zero momentum to a non-zero value. This is qualitatively consistent with other approaches. We also show that there is an enhancement of the propagator of one of the Zwanziger ghosts at two loops similar to that which occurs for the Faddeev-Popov ghosts. From the exact evaluation of the form factors we examine power corrections for the gluon propagator and the effective coupling. We find that both have the same qualitative behaviour in that the leading power correction is $O(1/p^2)$ and not $O(1/(p^2)^2)$.

Keywords: QCD, Renormalization Regularization and Renormalons, Confinement.

Contents

1. Introduction.

In the late seventies, Gribov pointed out that fixing a gauge in a non-abelian gauge theory was a non-trivial exercise in comparison with an abelian theory, [1]. In particular the gauge field was not fixed uniquely since for some gauge fields one could construct at least one other copy which could be obtained from the first by a gauge transformation. To understand this problem and the relation to confinement, Gribov restricted the region of integration in the path integral defining the non-abelian field theory to the region bounded by the *first* zero of the Faddeev-Popov operator, [1]. The boundary of this region, known as the Gribov horizon, defines the Gribov volume and is parametrised by the quantity γ, of mass dimension one, known as the Gribov mass. Though inside the Gribov horizon one can still have gauge copies, [2–4]. However, within this horizon is the fundamental modular region where each gauge field is uniquely defined, without copies, upon gauge fixing. Moreover, it had been suggested, [5], that any additional gauge copies inside the Gribov horizon do not affect the vacuum expectation values of any operator. Subsequent zeroes of the Faddeev-Popov operator beyond the first, define further regions of configuration space but it has been shown, [1], that they are equivalent to the first Gribov region which includes the origin. If one accepts that quantum chromodynamics (QCD), which is clearly a non-abelian gauge theory, has to be restricted to the volume defined by the Gribov horizon then Gribov argued that this had a profound effect on the infrared structure of the field theory, [3, 6–11]. For instance, in the Landau gauge Gribov showed that the ghost propagator diverged as $\frac{1}{(p^2)^2}$ as $p^2 \to 0$ rather than the usual $\frac{1}{p^2}$ behaviour which occurs in the ultraviolet (and perturbative) region, [1]. This property, known as ghost enhancement, followed directly as a result of the Gribov mass satisfying a gap equation which was determined at one loop explicitly, [1]. A comprehensive introduction to the original Gribov article is provided in [12].

In recent years, given the advance in non-perturbative techniques such as Dyson-Schwinger equations (DSE), the behaviour of the ghost propagator has been studied in the Landau gauge in the infrared and is in agreement with Gribov's enhancement observation, [13 – 16]. Though the actual power law exponent differs from that of Gribov, [13 – 16]. Further, such studies have opened up the possibility of examining other infrared features. For instance, it is believed the gluon propagator does not diverge at zero momentum, as its ultraviolet form would suggest, but either tends to zero or a finite value. Whilst Gribov demonstrated the gluon propagator vanished at the tree or classical level in the Landau gauge, [1], there appears to be no definitive agreement which of the latter scenarios persist in the quantum theory. For instance, a recent lattice study argues that the data imply a finite non-zero value for the gluon propagator at zero momentum, [17]. Whilst stated in these general terms it is worth bearing in mind that there are technical considerations such as gauge variance which may result in different pictures, when viewed in different calculational approaches and renormalization schemes. Though we note that throughout, our remarks will always be concerned with the Landau gauge. Equally controversial is the behaviour of the strong coupling constant $\alpha_S(p^2)$. Phenomenologically it is believed that $\alpha_S(p^2)$ freezes out at zero momentum to a finite non-zero value, [18 – 28]. Though as we will illustrate later what its precise value is appears to depend on the method used to extract it from experimental data as well as how one chooses to define it. More theoretical predictions based on properties of the underlying quantum field theory itself such as analyticity arguments or DSE, rather than the fits to experimental data, favour a value higher than those from data, [29 – 33]. Again these comments need to be tempered with the caveat that theoretical predictions are made as a consequence of certain approximations and in renormalization schemes which are mass dependent, which imply a gauge *dependent* coupling constant. However, as the quantity measured in DSE and lattice studies is a renormalization group invariant in the Landau gauge, which is measured at zero momentum, the hope is that such approximations do not significantly affect the estimates. Though some lattice and DSE studies do not see a finite freezing but instead observe an effective coupling which vanishes at zero momentum, [34 – 36]. Whether this is due to finite volume effects as argued in [33] by DSE studies, but subsequently rebutted in [37] is still not clear. Moreover, it would seem that one of the issues in this technical debate centres on the validity and application of the underlying Slavnov-Taylor identities to the fully non-perturbative régime which governs the infrared behaviour. Though it is worth noting that an effective coupling can be defined from any of the vertices of the QCD Lagrangian and the infrared behaviour of each does not have to be the same.

In the context of the Gribov approach to the infrared structure the advance made by Zwanziger, [7, 8, 10, 11], in reformulating the Gribov path integral and its inherent non-locality deriving from the limitation to the Gribov region is significant. In particular Zwanziger constructed a localized Lagrangian which involved extra ghost fields (which

JHEP05(2006)052

we will refer to as Zwanziger ghosts in contradistinction to the Faddeev-Popov ghosts), which when eliminated by their equations of motion reproduces the Gribov formulation of the problem. As of nearly equal significance is the fact that this Lagrangian (which we will refer to as the Gribov-Zwanziger Lagrangian) is renormalizable, [10, 38, 39], and explicitly contains the Gribov mass γ. This former property has been analyzed through the algebraic renormalization machinery and the Slavnov-Taylor identities have been established, [38, 39]. Interestingly despite the presence of the extra fields and a mass, no new independent renormalization constants are required in the Landau gauge to render the Gribov-Zwanziger Lagrangian finite. Moreover, as is evident in QCD when massive quarks are present the $\overline{\mathrm{MS}}$ renormalization of the coupling constant, gluon and Faddeev-Popov ghost fields remain unaffected by a non-zero γ, [38, 39]. Crucially the general formalism implies that the Gribov-Zwanziger Lagrangian can be used for performing loop calculations. Indeed Zwanziger, [8], reproduced the one loop gap equation of [1] by evaluating the vacuum expectation value of the Gribov mass operator which is equivalent to the implementation of the Gribov horizon condition. More recently this has been examined at two loops in [40] where the two loop correction to Gribov's gap equation has been determined in the Landau gauge with massless quarks in the $\overline{\mathrm{MS}}$ scheme. As a check that the gap equation is consistent, the two loop correction to the Faddeev-Popov ghost propagator was computed and shown that the ghost propagator enhancement is preserved as a $\frac{1}{(p^2)^2}$ behaviour precisely as a result of the two loop gap equation, [40].

Thus armed with a renormalizable localized Lagrangian, which at two loops appears to reproduce the observed behaviour noted by Gribov, we can attack the problem of the behaviour of other quantities in the infrared which we have discussed above. This is the aim of this article where we will compute the one loop correction to the gluon, Gribov-Zwanziger and Faddeev-Popov ghost propagators exactly. As will be evident this is not the straightforward exercise it would normally be in the absence of the Gribov-Zwanziger ghosts or the Gribov mass. However, we will be able to study the $p^2 \to 0$ behaviour of the gluon propagators and see if it vanishes at the one loop level or not. Equally worth studying is the effective coupling constant freezing at zero momentum. Since lattice and DSE studies examine an effective coupling constant which is renormalization group invariant but which depends only on the gluon and Faddeev-Popov ghost form factors, we will be able to construct this quantity and then take the $p^2 \to 0$ limit. We believe this will be constructive for reasons beyond the qualitative calculation we will perform. Specifically, previous studies have primarily been numerical and deduced either from experiment or by solving QCD with lattice or DSE techniques. The actual mechanism and which of the quantum fields truly drive the freezing have not really been studied. Having a Lagrangian where the calculations can be constructed explicitly will, we believe, be important in furthering that understanding. Given that we evaluate the propagators as *exact* functions of γ and the momentum, we can also study the corrections to the effective coupling in the alternative limit of $\gamma^2 \to 0$. The motivation for this is to examine whether one can not only produce power corrections but also to see whether such corrections are $O(1/p^2)$ as suggested by the lattice computation of [43, 44] or $O(1/(p^2)^2)$ as would be expected on the grounds of a condensation of the gauge invariant operator $G^a_{\mu\nu} G^{a\,\mu\nu}$. Finally, in referring to the work we present as qualitative, it is worth briefly clarifying at the outset that what we mean by this is that performing *one* loop calculations, where behaviour in the infrared appears to be consistent with non-perturbative highly intensive DSE and lattice studies, our analysis should only be regarded as comparative.

The paper is organised as follows. In section 2 we recall the main structure of the Gribov-Zwanziger Lagrangian and discuss the propagators and gap equation. The formal background to the computation of the one loop corrections to the 2-point functions of the Gribov-Zwanziger Lagrangian is introduced in section 3 prior to giving the details of the explicit computation in section 4. Section 5 is devoted to the problem of the freezing of the effective coupling constant in the $p^2 \to 0$ limit whilst we discuss the structure of the propagators and effective coupling constant in the other limit $\gamma^2 \to 0$ in section 6. Finally, our conclusions are given in section 7 and an appendix collects the full expressions for the exact 2-point functions.

2. Gribov-Zwanziger Lagrangian.

We begin by defining the localized Lagrangian derived by Zwanziger, [8], which replaces the non-local formulation of the gauge fixing condition of Gribov, [1], concentrating on the Landau gauge. In addition to the usual Faddeev-Popov ghosts, c^a and $\bar{c}^a$, of the canonical gauge fixing construction, the Gribov-Zwanziger Lagrangian involves additional Zwanziger ghosts. These fields, denoted by $\{\phi^{ab}_\mu, \bar{\phi}^{ab}_\mu\}$ and $\{\omega^{ab}_\mu, \bar{\omega}^{ab}_\mu\}$, are spin-1 and carry two colour indices. The latter set are anticommuting Grassmann fields whilst the former are commuting. Thus the full Gribov-Zwanziger Lagrangian is, [8],

$$\begin{aligned} L^{\mathrm{GZ}} &= L^{\mathrm{QCD}} + \bar{\phi}^{ab\,\mu}\partial^\nu \left(D_\nu\phi_\mu\right)^{ab} - \bar{\omega}^{ab\,\mu}\partial^\nu \left(D_\nu\omega_\mu\right)^{ab} \\ &\quad - g f^{abc}\partial^\nu\bar{\omega}^{ae}_\mu \left(D_\nu c\right)^b \phi^{ec\,\mu} - \frac{\gamma^2}{\sqrt{2}}\left(f^{abc}A^{a\,\mu}\phi^{bc}_\mu + f^{abc}A^{a\,\mu}\bar{\phi}^{bc}_\mu\right) + \frac{dN_A\gamma^4}{2g^2} \end{aligned} \qquad (2.1)$$

with

$$L^{\mathrm{QCD}} = \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} - \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 - \bar{c}^a\partial^\mu D_\mu c^a + i\bar{\psi}^{iI}\not{D}\psi^{iI} \qquad (2.2)$$

where A^a_μ is the gluon, ψ^{iI} is the (massless) quark and the indices lie in the ranges $1 \leq a \leq N_A$, $1 \leq i \leq N_F$ and $1 \leq I \leq N_f$ where N_F and N_A are the respective dimensions of the fundamental and adjoint representations and N_f is the number of quark flavours. The covariant derivatives are given by

$$\begin{aligned} D_\mu c^a &= \partial_\mu c^a - g f^{abc}A^b_\mu c^c \ , \quad D_\mu\psi^{iI} = \partial_\mu\psi^{iI} + igT^aA^a_\mu\psi^{iI} \\ (D_\mu\phi_\nu)^{ab} &= \partial_\mu\phi^{ab}_\nu - g f^{acd}A^c_\mu\phi^{db}_\nu \ . \end{aligned} \qquad (2.3)$$

For completeness, we have included the covariant gauge fixing parameter α which is in principle required in the derivation of the gluon propagators. The Gribov mass, γ, is present in the 2-point term mixing between the gluon and the Zwanziger ghosts. In various articles by other authors the factor of $\frac{1}{\sqrt{2}}$ is absent. However, as we will observe later this choice is essential to retaining a gluon propagator which is the same form as that originally derived by Gribov, [1]. Though, we note now that changing the numerical coefficient of

γ does not alter the overall physics but merely rescales the value of the gluon mass as a function of γ. The presence of the mixed 2-point term is problematic from the point of view of developing Feynman rules since one would have a mixed gluon Zwanziger ghost propagator. One way to circumvent this would be to redefine the A^a_μ and ϕ^{ab}_μ fields in such a way that the 2-point term is absent. It transpires that this is cumbersome to implement and in earlier work, [40], an approach has been found which allows one to systematically evaluate Feynman diagrams with a mixed propagator and to derive the two loop correction to the Gribov mass gap.

To derive the Feynman rules from (2.1) for the vertices of the interaction Lagrangian is straightforward. However, the main difficulty is to determine the propagator for the gluon in the presence of the mixing term. Although this was described in [40] it is instructive to develop the derivation since it serves as the basis for deducing the one loop correction to the gluon form factor. First, we note that in general in deriving the propagators from the quadratic part of a Lagrangian the fields are first transformed to momentum space. Therefore, concentrating only on the gluon and Zwanziger ghost part of (2.1) we have

$$L^{\rm GZ}_{\rm quad} = \frac{1}{2} A^a_\mu(-p) \left[p^2 \eta^{\mu\nu} - \left(1 - \frac{1}{\alpha} \right) p^\mu p^\nu \right] A^a_\nu(p) - \frac{\gamma^2}{\sqrt{2}} f^{abc} A^a_\mu(-p) \phi^{bc\,\mu}(p) + \frac{\gamma^2}{\sqrt{2}} f^{abc} \bar{\phi}^{ab\,\mu}(-p) A^c_\mu(p) - p^2 \bar{\phi}^{ab\,\mu}(-p) \phi^{ab}_\mu(p) \, . \tag{2.4}$$

If γ was zero then the inversion of the momentum space operator associated with the terms quadratic in the fields proceeds in a textbook manner. However, for γ non-zero we must first write $L^{\rm GZ}_{\rm quad}$ in matrix form with respect to the basis $\left\{ \frac{1}{\sqrt{2}} A^a_\mu, \phi^{ab}_\mu \right\}$. The inclusion of the factor $\frac{1}{\sqrt{2}}$ into the basis is crucial since $L^{\rm GZ}_{\rm quad}$ clearly now becomes

$$\left(\frac{A^a_\mu(-p)}{\sqrt{2}}, \bar{\phi}^{ab}_\mu(-p) \right) \begin{pmatrix} \left[p^2 \eta^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right) p^\mu p^\nu \right] \delta^{ac} & -\gamma^2 f^{acd} \eta^{\mu\nu} \\ -\gamma^2 f^{cab} \eta^{\mu\nu} & -p^2 \eta^{\mu\nu} \delta^{ac} \delta^{bd} \end{pmatrix} \begin{pmatrix} \frac{A^c_\nu(p)}{\sqrt{2}} \\ \phi^{cd}_\nu(p) \end{pmatrix} \, . \tag{2.5}$$

Inverting the matrix using the unit matrix

$$\begin{pmatrix} \eta_{\mu\nu} \delta^{ab} & 0 \\ 0 & \eta_{\mu\nu} \delta^{ac} \delta^{bd} \end{pmatrix} \tag{2.6}$$

then the α dependent propagators are

$$\begin{aligned} \langle A^a_\mu(p) A^b_\nu(-p) \rangle &= -\delta^{ab} p^2 \left[\frac{P_{\mu\nu}(p)}{[(p^2)^2 + C_A \gamma^4]} + \frac{\alpha L_{\mu\nu}(p)}{[(p^2)^2 + \alpha C_A \gamma^4]} \right] \\ \langle A^a_\mu(p) \bar{\phi}^{bc}_\nu(-p) \rangle &= -\frac{f^{abc} \gamma^2}{\sqrt{2}} \left[\frac{P_{\mu\nu}(p)}{[(p^2)^2 + C_A \gamma^4]} + \frac{\alpha L_{\mu\nu}(p)}{[(p^2)^2 + \alpha C_A \gamma^4]} \right] \\ \langle \phi^{ab}_\mu(p) \bar{\phi}^{cd}_\nu(-p) \rangle &= -\frac{\delta^{ac} \delta^{bd}}{p^2} \eta_{\mu\nu} \\ &\quad + \frac{f^{abe} f^{cde} \gamma^4}{p^2} \left[\frac{P_{\mu\nu}(p)}{[(p^2)^2 + C_A \gamma^4]} + \frac{\alpha L_{\mu\nu}(p)}{[(p^2)^2 + \alpha C_A \gamma^4]} \right] \end{aligned} \tag{2.7}$$

JHEP05(2006)052

where

$$P_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \quad , \quad L_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2} \tag{2.8}$$

are the respective transverse and longitudinal projectors and we have been careful to restore the basis vector $\left\{ \frac{1}{\sqrt{2}} A^a_\mu, \phi^{ab}_\mu \right\}$ and its transpose prior to deducing (2.7) which is the origin of the location of the factor of $\frac{1}{\sqrt{2}}$ in the mixed propagator. Further, without the factor of $\frac{1}{\sqrt{2}}$ in the mixed term of (2.1) one would not only have $\sqrt{2}\gamma^2$ in the off-diagonal terms of the matrix of (2.5) but also one would have a common factors of $[(p^2)^2 + 2C_A\gamma^4]$ and $[(p^2)^2 + 2\alpha C_A\gamma^4]$ in the propagators. In the Landau gauge calculation of [1] this additional numerical factor of 2 was absent and we choose to be consistent with [1] as our convention. Moreover, as we will later have to write $[(p^2)^2 + C_A\gamma^4]$ as the product of the more conventional factors $[p^2 + i\sqrt{C_A}\gamma^2]$ and $[p^2 - i\sqrt{C_A}\gamma^2]$ the choice in (2.1) therefore also turns out to be the most convenient. Next we note that in the Landau gauge the final propagators for the gluon and Zwanziger ghosts are, [8],

$$\begin{aligned}
\langle A^a_\mu(p) A^b_\nu(-p) \rangle &= - \frac{\delta^{ab} p^2}{[(p^2)^2 + C_A\gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \bar{\phi}^{bc}_\nu(-p) \rangle &= - \frac{f^{abc}\gamma^2}{\sqrt{2}[(p^2)^2 + C_A\gamma^4]} P_{\mu\nu}(p) \\
\langle \phi^{ab}_\mu(p) \bar{\phi}^{cd}_\nu(-p) \rangle &= - \frac{\delta^{ac}\delta^{bd}}{p^2}\eta_{\mu\nu} + \frac{f^{abe} f^{cde}\gamma^4}{p^2[(p^2)^2 + C_A\gamma^4]} P_{\mu\nu}(p) \ .
\end{aligned} \tag{2.9}$$

Whilst this may be an unusual way of considering the quadratic terms we note that it is primarily forced, aside from the original mixing, by the fact that the gluon is a real field whereas the Zwanziger ghosts are complex. Finally, for completeness we note that the propagators of the remaining fields are

$$\begin{aligned}
\langle c^a(p) \bar{c}^b(-p) \rangle &= - \frac{\delta^{ab}}{p^2} , \qquad \langle \psi^{iI}(p) \bar{\psi}^{jJ}(-p) \rangle = i\delta^{ij}\delta^{IJ} \frac{\not{p}}{p^2} \\
\langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle &= - \frac{\delta^{ac}\delta^{bd}\eta_{\mu\nu}}{p^2} \ .
\end{aligned} \tag{2.10}$$

Having summarized the derivation of the mixed propagators we note that the ϕ^{ab}_μ fields are the ones which implement the Gribov horizon condition, [8]. To recap from (2.1) the equation of motion of $\bar{\phi}^{ab}_\mu$ gives

$$\phi^{ab}_\mu = \frac{\gamma^2}{\sqrt{2}} f^{abc} \frac{1}{\partial^\nu D_\nu} A^c_\mu \tag{2.11}$$

whence one has

$$f^{abc} \langle A^{a\,\mu}(x) \phi^{bc}_\mu(x) \rangle = \frac{dN_A\gamma^2}{\sqrt{2}g^2} \tag{2.12}$$

where γ is defined by the horizon condition, [8],

$$\left\langle A^a_\mu \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} \right\rangle = \frac{dN_A}{C_A g^2} \ . \tag{2.13}$$

JHEP05(2006)052

In noting this then it is elementary to reproduce Gribov's original gap equation by integrating the mixed propagator over the loop momentum p of (2.9) to give the $\overline{\text{MS}}$ result, [1, 8],

$$1 = C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a + O(a^2) \tag{2.14}$$

where $a = g^2/(16\pi^2)$ and μ is the renormalization scale introduced to ensure that the coupling constant remains dimensionless in d dimensions when dimensional regularization is used. We have also absorbed the usual factor of $4\pi e^{-\bar{\gamma}}$ into μ^2 where $\bar{\gamma}$ is the Euler-Mascheroni constant. Although the gap equation is divergent the infinities are absorbed in the renormalization constants defined by

$$\begin{aligned} A^{a\,\mu}_{\text{o}} &= \sqrt{Z_A}\, A^{a\,\mu} \,, \quad c^a_{\text{o}} = \sqrt{Z_c}\, c^a \,, \quad \phi^{ab}_{\mu\,\text{o}} = \sqrt{Z_\phi}\, \phi^{ab}_\mu \,, \quad \omega^{ab}_{\mu\,\text{o}} = \sqrt{Z_\omega}\, \omega^{ab}_\mu \\ \psi_{\text{o}} &= \sqrt{Z_\psi}\psi \,, \quad g_{\text{o}} = Z_g\, g \,, \quad \gamma_{\text{o}} = Z_\gamma\, \gamma \end{aligned} \tag{2.15}$$

where $_{\text{o}}$ denotes a bare quantity and g and γ are the running coupling constant and running Gribov mass respectively. We note that it was shown in [38, 39] that in the Landau gauge

$$Z_c = Z_\phi = Z_\omega = \frac{1}{Z_g \sqrt{Z_A}} \tag{2.16}$$

and, in our conventions,

$$Z_\gamma = (Z_A Z_c)^{-1/4} \ . \tag{2.17}$$

Explicitly at three loops one has in $\overline{\text{MS}}$

$$\begin{aligned} \gamma_\gamma(a) = {} & [16 T_F N_f - 35 C_A] \frac{a}{48} + \left[280 C_A T_F N_f - 449 C_A^2 + 192 C_F T_F N_f \right] \frac{a^2}{192} \\ & + \Big[(486\zeta(3) - 75607) C_A^3 + (89008 - 15552\zeta(3)) C_A^2 T_F N_f \\ & \quad + (20736\zeta(3) + 19920) C_A C_F T_F N_f - 12352 C_A T_F^2 N_f^2 \\ & \quad - 3456 C_F^2 T_F N_f - 8448 C_F T_F^2 N_f^2 \Big] \frac{a^3}{6912} + O(a^4) \ . \end{aligned} \tag{2.18}$$

Though the four loop $\overline{\text{MS}}$ expression for the colour group SU(N) is available in [45], from knowledge of the Landau gauge gluon and Faddeev-Popov ghost anomalous dimensions.

In [1] the gap equation was used to show that in the infrared the ghost propagator is enhanced and behaves as $\frac{1}{(p^2)^2}$ as $p^2 \to 0$. This was obtained by computing the single one loop correction to the ghost 2-point function and noting that in the p^2 expansion of the diagram the leading p^2 correction was precisely the $O(a)$ part of (2.14) for all μ. More recently (2.1) has been used to compute the two loop correction to the horizon condition, (2.12), and the ghost 2-point function. In [40] the $\overline{\text{MS}}$ correction for massless quarks, is

$$\begin{aligned} 1 = {} & C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a \\ & + \Bigg[C_A^2 \Bigg(\frac{2017}{768} - \frac{11097}{2048} s_2 + \frac{95}{256} \zeta(2) - \frac{65}{48} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) + \frac{35}{128} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 \end{aligned}$$

$$+\frac{1137}{2560}\sqrt{5}\zeta(2)-\frac{205\pi^2}{512}\Bigg)$$

$$+C_AT_FN_f\left(-\frac{25}{24}-\zeta(2)+\frac{7}{12}\ln\left(\frac{C_A\gamma^4}{\mu^4}\right)-\frac{1}{8}\left(\ln\left(\frac{C_A\gamma^4}{\mu^4}\right)\right)^2+\frac{\pi^2}{8}\right)\Bigg]a^2$$

$$+O(a^3) \tag{2.19}$$

where $s_2 = (2\sqrt{3}/9)\mathrm{Cl}_2(2\pi/3)$, $\mathrm{Cl}_2(x)$ is the Clausen function and $\zeta(n)$ is the Riemann zeta function. Interestingly the $O(p^2)$ part of the $O(a^2)$ correction to the ghost 2-point function matched precisely with the $O(a^2)$ part of (2.19) for all μ so that in (2.1) the ghost propagator is still enhanced in the infrared limit. This is one of the conditions of the Kugo-Ojima confinement mechanism, [46], which ensures that the Gribov-Zwanziger Lagrangian is an important tool for exploring the infrared structure of QCD from an analytic point of view.

In the context of this Faddeev-Popov ghost enhancement at two loops in (2.1) we have also studied the structure of the 2-point function of the anticommuting Zwanziger ghost field ω^{ab}_μ in the same infrared limit. Although this is a spin-1 field it has some similarity with the corresponding Faddeev-Popov ghost field c^a since they both have the same wave function renormalization constant. It transpires that in evaluating the 1 one loop and 31 two loop Feynman diagrams contributing to the ω^{ab}_μ 2-point function the same phenomenon occurs as the Faddeev-Popov ghost. The two loop gap equation, (2.19), emerges in the same way in the p^2 expansion of the 2-point function so that the ω^{ab}_μ ghost is also enhanced in the infrared. In other words as $p^2 \to 0$ the propagator behaves as $\frac{1}{(p^2)^2}$ rather than having the $\frac{1}{p^2}$ behaviour of the initial propagator. It is worth remarking that as far as we are aware the discussion of [46] centred on a theory which involved only Faddeev-Popov ghosts and not the extension with the extra Zwanziger ghosts. Whilst in principle one would have to revisit that analysis to establish that the Kugo-Ojima mechanism still holds for (2.1), the fact that the ω^{ab}_μ propagator has precisely the same structure of the Faddeev-Popov ghost would seem to imply that the Kugo-Ojima confinement criterion is still valid for (2.1). Indeed it is worth observing that in the specific calculation, from the form of the ω^{ab}_μ propagator and its interaction with the gluon, the Lorentz structure of the ω^{ab}_μ 2-point function actually factors off and effectively leaves the Faddeev-Popov ghost 2-point function.

3. Formal derivation of propagators.

We now turn to the computation of the one loop corrections to the gluon propagator. However, given the nature of the problem we will also deduce information on the mixed propagator and ϕ^{ab}_μ propagator simultaneously. First, we recall the normal procedure to determine loop corrections to a field where there is no 2-point mixing. Essentially one computes the corrections to the 2-point function and then inverts this expression truncating at the order in the coupling constant one is interested in. For (2.1) the procedure is the same. However, one is dealing with a 2 × 2 matrix and has to first determine the

JHEP05(2006)052

corrections to each 2-point function which comprise the elements of the matrix. Once this has been achieved then the matrix can be inverted and all the propagators deduced from the appropriate elements. Several comments are worth making at this point. If one considers the location of the gluon propagator in the inverted matrix of 2-point functions its structure is derived from two pieces. One is the determinant of the matrix which by definition involves all the elements of the original matrix and which will contribute to the denominator of the gluon propagator. The other piece will be located in the numerator of the final propagator correction and will also involve pieces from the Zwanziger ghost ϕ^{ab}_μ 2-point function. In other words the fields implementing the horizon condition will form a central, and as will be seen later, a crucial role in the behaviour of the gluon propagator and effective coupling constant in the infrared. It is worth recalling that these properties only result as a consequence of a non-zero γ and hence the presence of a mixing term.

We have described our strategy in a general formal way at the outset since the practicalities of the inversion is technical. First, one is dealing with spin-1 fields which require a gauge fixing term to have a non-singular inversion in momentum space. As this is standard we note that one can either introduce a gauge fixing term as we did in the previous section before setting $\alpha = 0$ to specify the Landau gauge we are interested in, or one can work in the Landau gauge directly and factor off the common transverse projector $P_{\mu\nu}(p)$ from the 2-point matrix. We choose to do the latter. However, another complication of the inversion lies in correctly taking account of the group theory structure of the ϕ^{ab}_μ 2-point function corrections. For the gluon itself and the mixed 2-point functions the one loop corrections have the same group structure as the Lagrangian term. However, as the ϕ^{ab}_μ-field carries two group indices then its 2-point function will involve four colour index objects. If we denote these indices by the set $\{a, b, c, d\}$ then we take the basis of independent rank four objects which can arise from the one loop diagrams as

$$\left\{ \delta^{ac}\delta^{bd}, \delta^{ad}\delta^{bc}, \delta^{ab}\delta^{cd}, f^{ace}f^{bde}, f^{abe}f^{cde}, d_A^{abcd} \right\} \tag{3.1}$$

where the Jacobi identity excludes $f^{ade}f^{bce}$ from being independent. Though we could equally have chosen either of the other two combinations. The object d_A^{abcd} is defined as

$$d_A^{abcd} = \frac{1}{6}\mathrm{Tr}\left(T^a_A T^{(b}_A T^c_A T^{d)}_A \right) \tag{3.2}$$

where $(T^a)_{bc} = -\, i f^{abc}$ is the adjoint representation of the group generators and d_R^{abcd} is the totally symmetric trace of four group generators in the R representation, [47]. To proceed with the inversion of the mixed 2-point function this colour structure has to be taken into account in the ϕ^{ab}_μ 2-point function and its corresponding inverse element. As this is a formal discussion we will not cloud the inversion by including the explicit results of the loop integrals at this stage but take the general structure of the matrices we are interested in as follows. The matrix of 2-point functions at one loop is

$$\begin{pmatrix} p^2\delta^{ac} & -\gamma^2 f^{acd} \\ -\gamma^2 f^{cab} & -p^2\delta^{ac}\delta^{bd} \end{pmatrix} + \begin{pmatrix} X\delta^{ac} & Uf^{acd} \\ Mf^{cab} & Q\delta^{ac}\delta^{bd} + Wf^{ace}f^{bde} + Rf^{abe}f^{cde} + Sd_A^{abcd} \end{pmatrix} a + O(a^2) \tag{3.3}$$

JHEP05(2006)052

with respect to the basis $\left\{ \frac{1}{\sqrt{2}} A^a_\mu, \phi^{ab}_\mu \right\}$ where we have factored off the Lorentz structure. The quantities X, U, M, Q, W, R and S represent the one loop corrections. Similarly the Landau gauge inverse will be of the form

$$\begin{pmatrix} \frac{p^2}{[(p^2)^2+C_A\gamma^4]}\delta^{cp} & -\frac{\gamma^2}{[(p^2)^2+C_A\gamma^4]} f^{cpq} \\ -\frac{\gamma^2}{[(p^2)^2+C_A\gamma^4]} f^{pcd} & -\frac{1}{p^2}\delta^{cp}\delta^{dq} + \frac{\gamma^4}{p^2[(p^2)^2+C_A\gamma^4]} f^{cdr} f^{pqr} \end{pmatrix}$$
$$+ \begin{pmatrix} A\delta^{cp} & C f^{cpq} \\ E f^{pcd} & G\delta^{cp}\delta^{dq} + J f^{cpe} f^{dqe} + K f^{cde} f^{pqe} + L d_A^{cdpq} \end{pmatrix} a + O(a^2) \qquad (3.4)$$

where we have included the propagators from the previous section and the quantities A, C, E, G, J, K and L will depend on the one loop corrections defined in (3.3). Given these forms it is straightforward to check that

$$A = -\frac{1}{[(p^2)^2 + C_A\gamma^4]^2} \left[(p^2)^2 X - C_A\gamma^2 p^2 U - C_A\gamma^2 p^2 M + C_A\gamma^4 \left(Q + C_A R + \frac{1}{2} C_A W \right) \right]$$
$$C = \frac{1}{[(p^2)^2 + C_A\gamma^4]^2} \left[\gamma^2 p^2 X - C_A\gamma^4 M + (p^2)^2 U - \gamma^2 p^2 \left(Q + C_A R + \frac{1}{2} C_A W \right) \right]$$
$$E = \frac{1}{[(p^2)^2 + C_A\gamma^4]^2} \left[\gamma^2 p^2 X - C_A\gamma^4 U + (p^2)^2 M - \gamma^2 p^2 \left(Q + C_A R + \frac{1}{2} C_A W \right) \right]$$
$$G = -\frac{Q}{(p^2)^2} \quad , \quad J = -\frac{W}{(p^2)^2} \quad , \quad L = -\frac{S}{(p^2)^2}$$
$$K = -\frac{1}{[(p^2)^2 + C_A\gamma^4]^2} \left[\gamma^4 X + \gamma^2 p^2 U + \gamma^2 p^2 M + (p^2)^2 R - \gamma^4 \left(Q + \frac{1}{2} C_A W \right) \right]$$
$$+ \frac{\gamma^4}{(p^2)^2[(p^2)^2 + C_A\gamma^4]} \left[Q + \frac{1}{2} C_A W \right] \qquad (3.5)$$

by ensuring that the $O(a)$ term of the product of (3.3) and (3.4) vanishes. As a check on this inversion we note that $C = E$ if $M = U$. Also in determining the $O(a)$ correction to the mixed propagator of (2.7) C and E have to be divided by $\sqrt{2}$ as was noted after (2.8). We have derived these corrections at the formal level to indicate how involved the one loop corrections to the propagators are. However, given the intricate relation with the 2-point function corrections it will transpire that they will only be of use at this formal level.

4. One loop corrections.

We now detail the computation of the one loop integrals. First, the required Feynman diagrams are generated with the use of the QGRAF package, [48]. For the gluon and ϕ^{ab}_μ 2-point functions there are eight and two diagrams respectively whilst there are two diagrams contributing to either of the mixed 2-point functions giving a total of fourteen. Although we are primarily interested in examining the $p^2 \to 0$ limit of the propagators to ascertain whether the one loop gluon propagator still vanishes as it does at one loop, we have decided to evaluate the one loop correction *exactly* as a function of p^2. This requires the explicit evaluation of the one type of master integral which is of the form

$$I_1(p, m_1^2, m_2^2; \alpha, \beta) = \int_k \frac{1}{[k^2 + m_1^2]^\alpha [(k-p)^2 + m_2^2]^\beta} \qquad (4.1)$$

JHEP05(2006)052

where at one loop α and β are integers and the mass arguments take any combination of values in the set $\{0, i\sqrt{C_A}\gamma^2, -i\sqrt{C_A}\gamma^2\}$. This form, (4.1), derives from rewriting the propagators with the Gribov structure $p^2/[(p^2) + C_A\gamma^4]$ using partial fractions such as

$$\frac{p^2}{[(p^2)^2 + C_A\gamma^4]} = \frac{1}{2}\left(\frac{1}{[p^2 + i\sqrt{C_A}\gamma^2]} + \frac{1}{[p^2 - i\sqrt{C_A}\gamma^2]}\right) \tag{4.2}$$

producing standard propagators but with a width. However, it is worth noting that in any final expression we derive, the answer must be real given that the initial integrals are real which provides an internal check on the computation. On a practical note we record that the decomposition (4.2) is readily implemented in a computer algebra programme written in the symbolic manipulation language FORM, [49]. Indeed the complete calculation we describe here has been performed automatically with FORM.

Whilst the type of master integral, $I_1(p, m_1^2, m_2^2; 1, 1)$ has been studied and exploited many times we note that the key difference here is the presence of the complex mass. However, in the exact evaluation of our integrals we note that we use the formal results for $I_1(p, m_1^2, m_2^2; 1, 1)$ with real m_i^2 before analytically continuing to the values $\pm i\sqrt{C_A}\gamma^2$ we are interested in when the masses are non-zero. For completeness we note that the results for the two central integrals we use are, expanded to the finite parts,

$$\begin{aligned}
I_1(p, i\sqrt{C_A}\gamma^2, i\sqrt{C_A}\gamma^2; 1, 1) = {} & \frac{1}{\epsilon} + 2 - \ln\left(\frac{C_A\gamma^4}{\mu^4}\right) \\
& - \frac{1}{\sqrt{2}}\left[\left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} + 1\right]^{1/2}\left[\frac{1}{2}\ln\left(\frac{16C_A\gamma^4}{(p^2)^2}\right)\right.\right. \\
& \qquad - \frac{1}{2}\ln\left[1 + \sqrt{(1 + \frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
& \qquad \left.\left. - \ln\left[\left(1 + \sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2} - \sqrt{2}\right]\right]\right. \\
& + \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1\right]^{1/2} \\
& \qquad \times \tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1\right]^{-1/2}\right] \\
& - i\left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} + 1\right]^{1/2} \\
& \qquad \times \tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1\right]^{-1/2}\right] \\
& + i\left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1\right]^{1/2}\left[\frac{1}{2}\ln\left(\frac{16C_A\gamma^4}{(p^2)^2}\right)\right.
\end{aligned}$$

JHEP05(2006)052

$$
-\frac{1}{2}\ln\left[1+\sqrt{(1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \tag{4.3}
$$

$$
-\ln\left[\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]\right]\right]+O(\epsilon)
$$

and its conjugate, and

$$
\begin{aligned}
I_1(p,i\sqrt{C_A}\gamma^2,-i\sqrt{C_A}\gamma^2;1,1) &= \frac{1}{\epsilon}+2-\frac{[p^2-2i\sqrt{C_A}\gamma^2]}{2p^2}\ln\left(\frac{i\sqrt{C_A}\gamma^2}{\mu^2}\right)\\
&-\frac{[p^2+2i\sqrt{C_A}\gamma^2]}{2p^2}\ln\left(\frac{-i\sqrt{C_A}\gamma^2}{\mu^2}\right)\\
&-\frac{\sqrt{[4C_A\gamma^4-(p^2)^2]}}{p^2}\tan^{-1}\left[-\frac{\sqrt{[4C_A\gamma^4-(p^2)^2]}}{p^2}\right]\\
&+O(\epsilon)\ .
\end{aligned} \tag{4.4}
$$

When one of the masses m_i^2 is zero, we use the result

$$
\begin{aligned}
\int_k \frac{1}{k^2[(k-p)^2+i\sqrt{C_A}\gamma^2]} &= \frac{1}{\epsilon}+2-\ln\left(\frac{i\sqrt{C_A}\gamma^2}{\mu^2}\right)\\
&-\frac{[p^2+i\sqrt{C_A}\gamma^2]}{p^2}\ln\left[\frac{[p^2+i\sqrt{C_A}\gamma^2]}{i\sqrt{C_A}\gamma^2}\right]+O(\epsilon)
\end{aligned} \tag{4.5}
$$

and its conjugate. Finally, for completeness the one loop vacuum bubble is

$$
\int_k \frac{1}{[k^2+i\sqrt{C_A}\gamma^2]^\alpha} = \frac{\Gamma(1-\frac{1}{2}d)}{(4\pi)^{d/2}}\left(i\sqrt{C_A}\gamma^2\right)^{\frac{1}{2}d-\alpha} \tag{4.6}
$$

which, together with its conjugate, is expanded in powers of ϵ and the resulting logarithms treated with

$$
\ln\left(i\sqrt{C_A}\gamma^2\right) = \frac{1}{2}\ln\left(C_A\gamma^4\right)+\frac{i\pi}{2} \tag{4.7}
$$

and its conjugate. We now discuss the 2-point functions themselves but first note that they have to be renormalized. These infinities are absorbed by the wave function renormalization constants given in section 2, and we follow the procedure used in [50] to remove them in an automatic calculation. Though we note that the mixed 2-point function is in fact finite as a consequence of the Slavnov-Taylor identity relating Z_γ to Z_A and Z_c. Given the rather involved form for the master integrals, (4.3) and (4.4), the full explicit expressions are large and do not serve to illustrate any major points which can be accessed by other methods we will discuss. Since we are primarily interested in the zero momentum limit, we will concentrate on extracting that behaviour. There are two ways of doing this. One is to expand the explicit expressions in powers of p^2 and truncate at the appropriate term. A second way is to return to the Feynman diagrams themselves and carry out the expansion of each integral in powers of p^2. This is known as the vacuum bubble expansion and is one method to renormalize a quantum field theory. Though of course applying this technique here would require the term which is finite in ϵ. We have chosen to do this calculation in addition to the exact calculation for two reasons. First, it provides an alternative check on the explicit result where the expansion of the exact result in powers of p^2 must agree with the vacuum bubble expansion. Second, if one were to go beyond one loop to carry out a two loop analysis, then the *exact* evaluation of the two loop integrals with masses in the set $\{0, i\sqrt{C_A}\gamma^2, -i\sqrt{C_A}\gamma^2\}$ would be a formidable task even if all the massive master integrals were known in closed form explicitly before analytic continuation. The vacuum bubble expansion provides a more practical and efficient approach. To proceed with the expansion, the propagators of the master integral $I_1(p, m_1^2, m_2^2; \alpha, \beta)$, for example, can be expanded in powers of p^2 where the p-dependent propagator is expanded recursively with

$$\frac{1}{[(k-p)^2+m^2]} = \frac{1}{[k^2+m^2]} + \frac{2kp-p^2}{[k^2+m^2][(k-p)^2+m^2]} \tag{4.8}$$

which is readily implemented in FORM. The truncation criterion is that the 2-point functions themselves are $O\left((p^2)^2\right)$. Given the fact that (2.1) is renormalizable then the $O\left((p^2)^2\right)$ terms will be ϵ-finite. However, in performing this bubble expansion we cannot apply the identity (4.8) when all the masses in a Feynman integral are zero. This is because such an integral has its momentum dependence predetermined by $(p^2)^\Gamma$ where Γ is the dimension of the integral. As the momentum dependence is fixed due to the masslessness of the graph one needs to first isolate such integrals prior to applying the vacuum bubble expansion to the remaining terms in the decomposition of the overall Feynman graph. Once the vacuum bubble expansion has been performed it is elementary to replace the vacuum bubbles with the general result (4.6).

Given this algorithm it is straightforward to implement it in FORM and we record the p^2 expansion of each of the 2-point functions. We find

$$\begin{aligned}
\langle A^a_\mu(-p) A^b_\nu(p)\rangle &= \delta^{ab}\left[p^2 + \left(\left(\frac{15}{32}\ln\left(\frac{p^2}{\mu^2}\right) + \frac{163}{192}\ln\left(\frac{C_A\gamma^4}{\mu^4}\right) - \frac{263}{144} + \frac{57\pi}{64}\frac{\sqrt{C_A}\gamma^2}{p^2}\right)C_A \right.\right.\\
&\qquad\qquad \left.\left. + \left(\frac{20}{9} - \frac{4}{3}\ln\left(\frac{p^2}{\mu^2}\right)\right) T_F N_f\right) p^2 a + O(a^2)\right] \\
&\quad + O\left((p^2)^2\right) \\
\langle A^a_\mu(-p) \bar{\phi}^{bc}_\nu(p)\rangle &= f^{abc}\left[1 + \left(\frac{31\pi}{192}\frac{\sqrt{C_A}p^2}{\gamma^2}\right) a + O(a^2)\right]\gamma^2 + O\left((p^2)^2\right) \\
\langle \phi^{ab}_\mu(-p) \bar{\phi}^{cd}_\nu(p)\rangle &= \left[\delta^{ac}\delta^{bd}\left[1 - \left(\frac{5}{8} - \frac{3}{8}\ln\left(\frac{C_A\gamma^4}{\mu^4}\right)\right) a\right] p^2 \right.\\
&\quad \left. + \frac{3}{64} f^{ace} f^{bde} p^2 a + \frac{1}{24} f^{abe} f^{cde} p^2 a + \frac{9}{32} d_A^{abcd}\frac{p^2}{C_A} a + O(a^2)\right] \\
&\quad + O\left((p^2)^2\right)
\end{aligned} \tag{4.9}$$

which determine X, M, U, Q, W, R and S. We note that the expansion using (4.8) agrees with the expansion of the explicit expressions on the right side of (4.3), (4.4) and (4.6). From (4.9) we can deduce several interesting properties of the $p^2 \to 0$ limit. First, it is clear that at one loop the gluon *propagator*, (3.5), vanishes as the momentum vanishes. This is because when one substitutes the explicit values for X, M, U, Q, W, R and S from (4.9) into the expression for A in (3.5) then one finds

$$A = \left[\frac{p^2}{C_A\gamma^4} \left[\frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) - \frac{215}{384} \right] a + O\left((p^2)^2 \right) \right] + O(a^2) \ . \tag{4.10}$$

The leading terms in this expression derive from only Q, R and W. This vanishing of the gluon propagator at one loop is then consistent with general expectations and lattice results. See, for instance, [51]. Further, if one examines the $\delta^{ac}\delta^{bd}$ channel of the ϕ^{ab}_μ propagator the one loop mass gap condition (2.14) alters the p^2 dependence in a way similar to what occurs in both the c^a and ω^{ab}_μ ghost propagators. Moreover, this is the channel which appears in the Lagrangian itself and this implies that if there was no complication from the mixing then there would be an enhancement in that channel.

Finally, we note that the *full* explicit expressions for the one loop corrections to the 2-point functions are recorded in appendix A. Whilst these can be substituted into the formal expressions for the propagators, (3.5), the explicit expressions do not lead to any further insight. Though the formal functional form may be of use in obtaining better parametrizations of lattice and DSE data in the low p^2 region. However, for the remaining fields which are the Faddeev-Popov ghost and the anticommuting Zwanziger ghost, the propagators are

$$\begin{aligned} D_c(p^2) = & \left[-1 + \left[\frac{5}{4} - \frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) + \frac{3\sqrt{C_A}\gamma^2}{4p^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right.\right. \\ & - \frac{3\pi\sqrt{C_A}\gamma^2}{8p^2} + \frac{C_A\gamma^4}{8(p^2)^2} \ln \left[1 + \frac{(p^2)^2}{C_A\gamma^4} \right] - \frac{3}{8} \ln \left[1 + \frac{(p^2)^2}{C_A\gamma^4} \right] \\ & \left.\left. - \frac{p^2}{4\sqrt{C_A}\gamma^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right] C_A a \right]^{-1} + O(a^2) \end{aligned} \tag{4.11}$$

where the ghost form factor is defined by

$$\langle c^a(p) \bar{c}^b(-p) \rangle = \frac{D_c(p^2)}{p^2} \delta^{ab} \ . \tag{4.12}$$

Using the gap equation, (2.14), we have

$$\begin{aligned} D_c(p^2) = & \left[\left[\frac{5}{8} + \frac{\pi p^2}{8\sqrt{C_A}\gamma^2} + \frac{3\sqrt{C_A}\gamma^2}{4p^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right.\right. \\ & - \frac{3\pi\sqrt{C_A}\gamma^2}{8p^2} + \frac{C_A\gamma^4}{8(p^2)^2} \ln \left[1 + \frac{(p^2)^2}{C_A\gamma^4} \right] \\ & \left.\left. - \frac{3}{8} \ln \left[1 + \frac{(p^2)^2}{C_A\gamma^4} \right] - \frac{p^2}{4\sqrt{C_A}\gamma^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right] C_A a \right]^{-1} + O(a^2) \ . \end{aligned} \tag{4.13}$$

So that $D_c^{-1}(p^2)$ is $O((p^2)^2)$ as $p^2 \to 0$. For ω^{ab}_μ it turns out that defining

$$\langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle = \delta^{ac}\delta^{bd} \frac{D_\omega(p^2)}{p^2} \eta_{\mu\nu} \tag{4.14}$$

then

$$D_\omega(p^2) = D_c(p^2) \tag{4.15}$$

to one loop and no longitudinal component is generated. So the ω^{ab}_μ ghost enhancement actually follows trivially from the equivalence with the Faddeev-Popov ghost for all momenta. This follows as a result of our earlier observation of the factoring of the Lorentz structure from the 2-point function. Given there is a similar enhancement at two loops from the vacuum bubble expansion, it is tempting to speculate that not only does (4.15) hold at two loops but maybe also to all orders. Whilst we have noted that there is a similar type of enhancement of Q in the ϕ^{ab}_μ-propagator, it is also turns out that $Q = -\left(D_c(p^2)\right)^{-1} = -\left(D_\omega(p^2)\right)^{-1}$ at one loop. Finally for the quark propagator, where the quarks are massless, defining the form factor by

$$\langle \psi^{iI}(p) \bar{\psi}^{jJ}(-p) \rangle = i \delta^{ij} \delta^{IJ} D_\psi(p^2) \frac{\not{p}}{p^2} \tag{4.16}$$

then

$$\begin{aligned} D_\psi(p^2) = & \left[1 + \left[\frac{1}{2} - \frac{C_A \gamma^4}{2(p^2)^2} \ln \left[1 + \frac{(p^2)^2}{C_A \gamma^4} \right] - \frac{3\sqrt{C_A}\gamma^2}{p^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right. \right. \\ & \left. \left. + \frac{3\pi}{4} \frac{\sqrt{C_A}\gamma^2}{p^2} - \frac{p^2}{2\sqrt{C_A}\gamma^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{p^2} \right] \right] C_F a \right]^{-1} \\ & + O(a^2) \ . \end{aligned} \tag{4.17}$$

We note that in the limit $p^2 \to 0$ then

$$D_\psi(p^2) = 1 + \left[\frac{3}{2} - \frac{\pi p^2}{4\sqrt{C_A}\gamma^2} + O(p^2) \right] C_F a + O(a^2) \tag{4.18}$$

so that the form factor tends to a constant at zero momentum.

5. α_S freeze-out.

Having examined the zero momentum limit of the 2-point functions and form factors, we now apply the results to the problem of the value of the effective strong coupling constant in the same limit. In both lattice and DSE analyses one can study the value of a renormalization group invariant effective coupling constant, denoted by $\alpha^{\mathrm{eff}}_S(p^2)$, which is defined as a result of the renormalization properties of the gluon ghost vertex in the Landau gauge. In terms of the gluon and Faddeev-Popov form factors, $D_A(p^2)$ and $D_c(p^2)$, where

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \delta^{ab} \frac{D_A(p^2)}{p^2} P_{\mu\nu}(p) \tag{5.1}$$

the effective coupling is

$$\alpha^{\mathrm{eff}}_S(p^2) = \alpha(\mu) D_A(p^2) \left(D_c(p^2) \right)^2 \tag{5.2}$$

where $\alpha(\mu)$ is the running strong coupling constant and $\alpha = g^2/(4\pi)$. Due to the gluon ghost Slavnov-Taylor identity, [52], there is no contribution from the vertex form factor.

JHEP05(2006)052

Thus, given that we have one loop expressions for $D_A(p^2)$ and $D_c(p^2)$ in the $p^2 \to 0$ limit, it is straightforward to examine the structure of (5.2) in the same limit.

First, if we consider the expression with the tree level form factors

$$D_A(p^2) = \frac{(p^2)^2}{[(p^2)^2 + C_A\gamma^4]} + O(a) \ , \quad D_c(p^2) = 1 + O(a) \tag{5.3}$$

then clearly

$$\alpha_S^{\text{eff}}(p^2) = \frac{\alpha(\mu)(p^2)^2}{[(p^2)^2 + C_A\gamma^4]} \tag{5.4}$$

which vanishes as $p^2 \to 0$. However, if one includes the one loop corrections then with

$$D_c(p^2) = \left[-1 + \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) - \left(\frac{\pi}{8} \frac{p^2}{\sqrt{C_A}\gamma^2} \right) + O(p^4) \right] C_A a + O(a^2) \right]^{-1} \tag{5.5}$$

where we have expanded the propagator to the next term in the momentum expansion, we have

$$\alpha_S^{\text{eff}}(0) = \lim_{p^2 \to 0} \left[\frac{\alpha(\mu) \left[1 + C_A \left(\frac{3}{8} \ln \left(\frac{C_A\gamma^4(\mu)}{\mu^4} \right) - \frac{215}{384} \right) a(\mu) \right] (p^2)^2}{C_A\gamma^4(\mu) \left[1 + C_A \left(\frac{3}{8} \ln \left(\frac{C_A\gamma^4(\mu)}{\mu^4} \right) - \frac{5}{8} + \frac{\pi p^2}{8\sqrt{C_A}\gamma^2(\mu)} \right) a(\mu) \right]^2} \right] . \tag{5.6}$$

In this expression we have expanded the denominator factor of $D_A(p^2)$ and included it in the $O(p^2)$ part of the numerator. Though in fact it will not contribute to the final value of $\alpha_S^{\text{eff}}(0)$ which is derived from the displayed numerator and denominator factors. Before taking the limit one can use Gribov's gap equation, (2.14), to enforce the ghost enhancement in the ghost form factor which leaves the denominator as $O(a^2)$. This coupling constant dependence in fact cancels with similar factors in the numerator. One comes from the $\alpha(\mu)$ in the definition and another one derives from the numerator factor when the gap equation is used. Unlike the cancellation in the ghost propagator to give ghost enhancement, in the numerator the numerical term at one loop does not match that of the gap equation and one is left with an $O((p^2)^2)$ term in the numerator. This momentum dependence then cancels the $(p^2)^2$ term on the denominator. Finally, the running Gribov mass parameter, $\gamma(\mu)$, also cancels with one factor of γ^4 deriving from the determinant of the 2-point functions and one from the p^2/γ^2 term of the expansion of the Faddeev-Popov ghost 2-point function. Since the running coupling constant and dimensionful quantities also cancel one is left with a pure number when $p^2 \to 0$. We find

$$\alpha_S^{\text{eff}}(0) = \frac{50}{3\pi C_A} \ . \tag{5.7}$$

Specifying various gauge groups of interest, we have

$$\alpha_S^{\text{eff}}(0) \Big|_{\text{SU}(3)} = 1.7684 \tag{5.8}$$

and

$$\alpha_S^{\text{eff}}(0) \Big|_{\text{SU}(2)} = 2.6526 \ . \tag{5.9}$$

JHEP05(2006)052

This is a novel property of the Gribov-Zwanziger Lagrangian. From studying the calculation in the way we have presented it, we note that the gap equation was essential in removing the tree terms of the form factors, as well as providing factors of $a(\mu)$ and $\gamma(\mu)$ which *precisely* cancel. Indeed the role of $\gamma(\mu)$ seems crucial since it provides the balancing dimensionality for the momentum which eventually vanishes. More significantly it is interesting to note that the cancellation between the numerator and denominator factors is between pieces which involve the two types of ghost. In other words the anticommuting Faddeev-Popov ghost and the commuting Zwanziger ghost, ϕ^{ab}_μ. The contribution from the former, which is required for the usual gauge fixing, is clearly demanded by the definition but the latter arises through the implementation of the Gribov horizon which is, of course, an infrared contribution originating from the ambiguity of the gauge fixing procedure.

Concerning the actual numerical value of $\alpha^{\rm eff}_S(0)$ in SU(3), (5.8), it is worth contrasting with estimates from other methods. From a survey of articles (which is by no means exhaustive) it appears that the non-zero values fall into three classes. These are when $\alpha^{\rm eff}_S(0)$ is deduced from experimental data (P), from numerical studies such as lattice or DSE and from more analytic approaches, (A). Currently there is not a consensus of values from these three divisions which are summarised in table 1 where the result of [18] is the central value of the range the authors quote. For instance, the DSE value differs from the phenomenology based approach by an order of magnitude; with analytic approaches, including this article, bridging between the two extremes. However, in this context we note that (5.7) should be viewed as a qualitative result rather than quantitative. First, the computation, whilst simple in its derivation, is only at one loop. Clearly there are higher order corrections to the form factors. Indeed it is not immediately apparent how the $O(a^2)$ corrections would conspire to cancel to leave a result independent of $a(\mu)$ and $\gamma(\mu)$ as was the case at leading order. However, it is also not clear whether comparison of $\alpha^{\rm eff}_S(0)$ from different methods has any meaning in the first instance since it corresponds to the evaluation of a quantity defined in perturbation theory but evaluated in the infrared limit. Though the lattice and DSE estimates are for the same quantity as (5.2) in the Landau gauge. Moreover, (5.2) depends on the ghost form factor which would not be an object immediately accessible experimentally. In this context it is worth commenting on a recent proposal of [32, 33] where an effective coupling constant is defined from other vertices in the Gribov-Zwanziger Lagrangian. For instance, it was argued that one could examine the form factor of the triple gluon vertex, $D_{AAA}(p^2)$, and derive an effective coupling with $D_A(p^2)$, defined with respect to some momentum configuration for the external gluons, with

$$\alpha^{\rm eff}_{S\,AAA}(p^2) = \alpha(\mu) D_{AAA}(p^2) \left(D_A(p^2) \right)^3 \tag{5.10}$$

where $D_{AAA}(p^2)$ is necessary due to the lack of a Slavnov-Taylor identity comparable to that of the gluon ghost vertex. To have a freezing of $\alpha^{\rm eff}_{S\,AAA}(p^2)$, given the behaviour of $D_A(p^2)$ established here, then $D_{AAA}(p^2)$ would have to diverge in such a way that the momentum dependence cancels in the zero momentum limit, [32, 33]. Whilst this is essentially a dimensional observation, [32, 33], it does not necessarily follow that the *same* numerical value for the frozen effective coupling would emerge. Given that we now

JHEP05(2006)052

$\alpha_S^{\rm eff}(0)$	Method	Reference
0.47	AP	[20, 21]
0.56	P	[18]
0.60	P	[24 – 26]
0.60	P	[19]
0.63	P	[22, 23]
0.82	P	[27, 28]
1.40	A	[29 – 31]
1.77	A	this study
2.97	DSE	[32, 33]

Table 1: Non-zero estimates of $\alpha_S^{\rm eff}(0)$.

have a renormalizable localized Lagrangian implementing the Gribov horizon, one natural question which could be considered is whether this behaviour for the triple gluon vertex could be determined in the $p^2 \to 0$ limit. Although such a calculation is beyond the scope of the current article, we note that on renormalizability grounds $D_{AAA}(p^2)$ would have to be finite as $p^2 \to 0$. Therefore, if one is to extract a value for $\alpha_{S\,AAA}^{\rm eff}(0)$, via the suggestion of [32, 33], it would seem to us that the behaviour is truly driven by some non-perturbative mechanism.

Although we have given a summary of the non-zero finite values for $\alpha_S^{\rm eff}(0)$ it should be noted that various lattice and DSE studies find a value of zero as $p^2 \to 0$, (see, for example, [34, 37]), for the effective coupling defined from the gluon and ghost form factors. Further, lattice and DSE studies of effective couplings defined from other vertices also find a zero value in this limit. For instance, see [35, 36]. Therefore, it would appear that currently there is no common view of the precise zero momentum behaviour. However, these latter studies are of the effective coupling defined from the triple gluon and quark gluon vertices respectively. So if our argument about the finiteness of $D_{AAA}(p^2)$ as $p^2 \to 0$ from (2.1) is valid, then the vanishing of $D_A(0)$ would imply that $\alpha_{S\,AAA}^{\rm eff}(0) = 0$ which would appear to be consistent with the analyses of this vertex, [35].

The situation for SU(2) is similar, though of course the only results available are from lattice and DSE studies. Briefly, in the same way that certain SU(3) lattice and DSE computations give similar non-zero values for $\alpha_S^{\rm eff}(0)$ the same is true for SU(2) with the common freeze-out value of $\alpha_S^{\rm eff}(0) = 5(1)$, [53], or 5.2, [54]. Whilst this is significantly larger than our value of 2.65 there is at least the consistent observation that our SU(2) value is larger than the SU(3) value. Though from the explicit expression this is primarily due to the denominator color factor in (5.7). Taking the ratio of the SU(2) and SU(3) values for $\alpha_S^{\rm eff}(0)$ from the DSE analyses one finds the ratio 1.75 in comparison with the ratio of 1.5 of (5.7).

Finally, in comparing (5.7) with DSE estimates it is worth noting that the latter are invariably computed in MOM schemes which are mass dependent renormalization schemes. By contrast (5.7) has been deduced in the $\overline{\rm MS}$ scheme which is a mass independent scheme. Whilst ultimately the definition of $\alpha_S^{\rm eff}(p^2)$ is a renormalization group invariant, if the

JHEP05(2006)052

quantity (5.2) has a meaning in the infrared then the full value will be scheme independent. Therefore, computing the two loop corrections to (5.7) would be useful to see if there is convergence to a higher value. In discussing some of the issues concerning the freezing from a theoretical point of view, it is worth noting the one loop study of [55] where the infrared behaviour of an effective coupling defined via the quark gluon vertex with massive quarks in an arbitrary covariant gauge in the MOM scheme. There the presence of the gauge parameter led to differing zero momentum behaviours indicative of the subtleties associated with analysing the infrared behaviour of a quantity which has only justifiable meaning in the ultraviolet régime.

6. Power corrections.

Having studied the effective coupling in the limit of zero momentum, using both the bubble expansion and the exact expressions for the 2-point functions, we can now examine the propagators and the couplings in another limit. In [43, 44], a numerical fit of the lattice computation of the effective coupling constant suggested that in a certain momentum range, the coupling deviated from the expected (perturbative) behaviour by a piece which could be parametrized by power corrections. Intriguingly these corrections were of the form $O(1/p^2)$ and not the expected $O(1/(p^2)^2)$ behaviour. In other words, [43, 44],

$$\alpha_S^{\mathrm{eff}}(p^2) = \alpha_S^{\mathrm{pert}}(p^2) + \frac{c_2}{p^2} + O\left(\frac{1}{(p^2)^2}\right) \ . \tag{6.1}$$

An $O(1/(p^2)^2)$ correction is motivated by the fact that to match the dimensionality of the momentum dependence one needs a dimension four quantity. Given the fact that the operator product expansion and sum rule studies of [56] suggest the condensation of the gauge invariant operator $G^a_{\mu\nu} G^{a\,\mu\nu}$, then there is an a priori clear candidate for the numerator which is $\langle G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$. However, in order to have an $O(1/p^2)$ correction, one would have to have a lower dimensional operator condensing. Since one such operator is $\frac{1}{2} A^a_\mu A^{a\,\mu}$, this has led to the suggestion that this operator condenses, [43], and consequently estimates for $\langle \frac{1}{2} A^a_\mu A^{a\,\mu} \rangle$ have been derived from the operator product expansion, [57], and the local composite operator method, [58]. Indeed it was observed in [59, 60] that the perturbative QCD vacuum was unstable and the vacuum expectation value of such operators, and its non-local gauge invariant generalization, would play a significant role in understanding the true vacuum. Also, the condensation of such a dimension two operator was already discussed in the Coulomb gauge in [61]. Clearly $\frac{1}{2} A^a_\mu A^{a\,\mu}$ is a gauge variant operator but the appearance of its vacuum expectation value in the fits of [43, 44] is neither unexpected nor inconsistent with this since the effective coupling constant is gauge dependent in mass dependent renormalization schemes such as the MOM scheme used in [43, 44]. As a consequence of these observations concerning the apparent existence of a non-zero value for $\langle \frac{1}{2} A^a_\mu A^{a\,\mu} \rangle$ there has been renewed interest in trying to understand the dynamical generation of a gluon mass. However, given that the Gribov mass, γ^2, can effectively reproduce a gluon mass, it is worth investigating whether one can *mimic* the power correction behaviour of [43, 44] from the Gribov-Zwanziger Lagrangian, (2.1).

JHEP05(2006)052

Therefore, we have examined the integrals used to construct the form factor corrections exactly and expanded them in powers of $\sqrt{C_A}\gamma^2$. Though we remark that in the expansion in which we work $\sqrt{C_A}\gamma^2 < p^2$, so that we need to rewrite one of the parts of an exact integral through the replacement

$$\sqrt{[4C_A\gamma^4 - (p^2)^2]}\tan^{-1}\left[-\frac{\sqrt{[4C_A\gamma^4 - (p^2)^2]}}{p^2}\right]$$
$$\mapsto \sqrt{[(p^2)^2 - 4C_A\gamma^4]}\ln\left[\frac{\left[p^2 + \sqrt{[(p^2)^2 - 4C_A\gamma^4]}\right]}{\left[p^2 - \sqrt{[(p^2)^2 - 4C_A\gamma^4]}\right]}\right] . \tag{6.2}$$

Therefore expanding the 2-point functions in powers of γ^2, produces

$$\begin{aligned} X &= \left[\left[\left(\frac{13}{6}\ln\left(\frac{p^2}{\mu}\right) - \frac{97}{36}\right)p^2 + \frac{3\pi\sqrt{C_A}\gamma^2}{8}\right]C_A \right.\\ &\quad \left. - \left[\frac{4}{3}\ln\left(\frac{p^2}{\mu}\right) - \frac{20}{9}\right]T_F N_f p^2 + O(\gamma^4)\right]a + O(a^2) \\ M &= U = \left[\frac{11C_A}{8}\gamma^2 + O(\gamma^4)\right]a + O(a^2) \\ Q &= \left[\left[-\left(1 - \frac{3}{4}\ln\left(\frac{p^2}{\mu}\right)\right)p^2 + \frac{3\pi\sqrt{C_A}\gamma^2}{8}\right]C_A + O(\gamma^4)\right]a + O(a^2) \\ W &= R = S = O(a^2) \end{aligned} \tag{6.3}$$

and

$$D_c(p^2) = \left[-1 + \left[1 - \frac{3}{4}\ln\left(\frac{p^2}{\mu^2}\right) - \frac{3\pi}{8}\frac{\sqrt{C_A}\gamma^2}{p^2} + O(\gamma^4)\right]C_A a + O(a^2)\right]^{-1} \tag{6.4}$$

where we have neglected the term beyond the first γ^2/p^2 correction in each case. For the gluon and Faddeev-Popov ghost the finite γ-independent pieces both agree with the result of the massless calculation.

Interestingly the gluon propagator correction is $O(\gamma^2)$ and not $O(\gamma^4)$ as might have been suggested from the original gluon propagator which is a function of γ^4. Although the correction is from the one loop term and involves a, it qualitatively introduces an effective mass for the gluon in this next to high energy limit. One comment concerning the choice of the sign of γ^2 is worth making. Eliminating the auxiliary scalar field ϕ^{ab}_μ from (2.1) means that (2.1) is an even function of γ^2. One could fix the sign of γ^2 by arguing that it has to produce a non-tachyonic effective propagator of the usual form from the binomial expansion. However, one could equally choose the mass to be tachyonic without upsetting the two loop gap equation and the ghost enhancement. Although this may not appear to be a physically sensible choice, we remark that the study of [62] suggested that introducing an effective tachyonic gluon mass into current correlators could reproduce experimental data more accurately compared to the case of no gluon mass. Whilst there did not appear to be any justification for such a tachyonic gluon in [62], the freedom of choosing the sign of γ^2, whilst still *retaining* the infrared properties already discussed, does appear appealing.

JHEP05(2006)052

In addition we can now examine the effective coupling constant, (5.2), in the same limit. Retaining only the $O(\gamma^2)$ piece of A from the 2-point functions, then

$$D_A(p^2) = 1 - \frac{3C_A\pi}{8}\frac{\sqrt{C_A}\gamma^2}{p^2}a + O\left(\frac{1}{(p^2)^2}\right) \ . \tag{6.5}$$

In expressing the power corrections in this way, we are treating it as an expansion in powers of $\sqrt{C_A}\gamma^2/p^2$ since the appropriate factor of C_A is associated with the Gribov mass. Thus from the definition, and regarding the γ independent piece as corresponding to $\alpha_S^{\mathrm{pert}}(p^2)$, then to one loop we have

$$\alpha_S^{\mathrm{eff}}(p^2) = \alpha_S^{\mathrm{pert}}(p^2)\left[1 - \frac{9C_A\pi}{8}\frac{\sqrt{C_A}\gamma^2}{p^2}a + O\left(\frac{1}{(p^2)^2}\right)\right] \tag{6.6}$$

in $\overline{\mathrm{MS}}$. From the $O(\gamma^2)$ corrections one *formally* produces an $O(1/p^2)$ term as the leading power correction as opposed to an $O(1/(p^2)^2)$ one. Also the coefficient has the opposite sign to that of [43, 44] and would be in keeping with our suggested choice of changing the sign of γ^2. However, since that calculation was in the MOM scheme in contrast to the $\overline{\mathrm{MS}}$ scheme *one* loop computation here, this would not necessarily be sufficient justification for altering the sign of γ^2. Again we merely regard it as an interesting observation. Further, we emphasise that, as in earlier sections, these are qualitative observations of this effective coupling constant deriving from a one loop calculation with the motivation being to appreciate the implications of the Gribov parameter in comparison with non-perturbative numerical analyses. Clearly such a power correction could equally well be produced by the presence of $\langle \frac{1}{2}A^a_\mu A^{a\,\mu}\rangle$ instead of the condensate (2.12). Indeed there have been more recent studies, [39, 63, 64], of the effect a non-zero $\langle \frac{1}{2}A^a_\mu A^{a\,\mu}\rangle$ has in the Gribov-Zwanziger Lagrangian in the Landau gauge. We also remark that from the renormalization structure, [38, 39], both $\frac{1}{2}A^a_\mu A^{a\,\mu}$ and γ^2 have the same anomalous dimensions. So that a combination of both these dimension two operator vacuum expectation values could be responsible for the $O(1/p^2)$ correction in the data of [43, 44]. At a more formal level it is worth commenting on the structure of the $O(\gamma^2)$ correction in (6.6). This effective coupling constant is by construction a renormalization group invariant quantity due to the underlying Slavnov-Taylor identity for this vertex, [65]. For the Gribov-Zwanziger Lagrangian this identity remains valid which can be observed, for instance, from explicit loop calculations of the relevant anomalous dimensions using (2.1) which reflect the ultraviolet position. As is evident from the freezing calculation, the numerical value of $\alpha_S^{\mathrm{eff}}(0)$ is clearly a renormalization group invariant. However, it is not immediately apparent if the power correction, which is manifest in the remaining energy range, also retains this feature. To fully examine this situation is not as straightforward as it would simply appear. This is primarily due to the fact that the full renormalization group for (2.1) has not yet been constructed. Ordinarily this involves parameters and quantities which are *independent*. By contrast, in (2.1) the Gribov mass, being a parameter of the theory which appears in the (ultraviolet) renormalization group equation, is not independent by virtue of the Gribov gap equation. Therefore, to fully access the renormalization group properties of quantities deduced from (2.1) it would seem necessary in the first instance to develop the renormalization group for (2.1) subject to the Gribov gap equation constraint. This is beyond the scope of this article.

If we also consider the quark propagator, then in the same limit we find

$$D_\psi(p^2) = 1 + \left[\frac{3\pi\sqrt{C_A}\gamma^2}{4p^2} + O\left(\frac{1}{(p^2)^2}\right) \right] C_F a + O(a^2) \ . \qquad (6.7)$$

By the same token that we argued that the power corrections appeared to generate a mass for the gluons one might regard this correction as a generated quark mass. Although the sign is opposite to what one observes in the gluon case but with magnitude $3\pi C_F a/4$, we emphasise that this is in the situation where there is no initial quark mass and therefore it does not correspond to the part of a fermionic propagator which ordinarily defines the mass term.

Finally, if one accepts that there can be an effect from the Gribov parameter in studying power corrections in gauge variant quantities, then considering higher power corrections leads to the conclusion that one could have contributions from not only $\langle G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$ but also γ^4 in the $O(1/(p^2)^2)$ correction. Therefore, in extracting estimates for $\langle G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$ in such calculations, one would in principle need to make allowance for the potential presence of additional γ^4 type terms. For the computation of gauge invariant quantities, there ought not to be any contribution from the Gribov mass.

7. Discussion.

In examining the one loop corrections to the propagators in the Gribov-Zwanziger Lagrangian several interesting features have emerged. First, we have shown that the gluon propagator vanishes in the $p^2 \to 0$ limit which is consistent with DSE and some lattice studies. More significantly we have seen to what extent the gap equation satisfied by the Gribov parameter underlies the infrared structure of the propagators and effective coupling constant in (2.1). In [1] it was used to ensure that the ghost propagator was enhanced in the infrared. However, here it has been used to show that the ω^{ab}_μ ghost propagator is also enhanced at two loops, and that the behaviour of the tree channel of the ϕ^{ab}_μ propagator is also altered in the infrared at one loop. More significantly the gap equation played a central role in the freezing of the renormalization group invariant effective coupling defined via the gluon Faddeev-Popov ghost vertex and has given some insight into how a non-zero finite value emerges. For instance, in the definition (5.2) if the ghost propagator does not satisfy the gap equation then the factors of p^2 in the zero momentum limit will not match between the numerator and denominator to leave a momentum independent quantity. Any remaining momentum dependence would dominate the $p^2 \to 0$ behaviour. Since there appears to be a discrepancy in various numerical approaches between a non-zero and zero value for $\alpha^{\mathrm{eff}}_S(0)$, it may be due to the Kugo-Ojima condition not being fully and *precisely* satisfied. Although our one loop calculation has led to a freeze-out value which is different from that from DSE and certain lattice studies, and is regarded as qualitative in the sense described earlier, it would be interesting to see what effect the two loop corrections have on the numerical estimate. At one loop the result is clearly independent of the number of quarks but quark loops will be present at two loops. Recently in [66] a comparison was made between a calculation of (5.2) in quenched QCD and one where dynamical fermions

JHEP05(2006)052

were used. Although both computations suggested $\alpha_S^{\rm eff}(0) = 0$, the peak of the plot of $\alpha_S^{\rm eff}(p^2)$ is lower in the dynamical case. Though we note that intriguingly the peak in the quenched case appears to be around 1.7 which is similar to (5.8). In concluding this aspect of our analysis it would be fair to make the general observation that the full resolution as to whether the quantity (5.2) freezes at a zero or non-zero value in the infrared has not yet been achieved and our value is merely another contribution to that debate.

Another aspect of this study has been to appreciate the role played by the extra Zwanziger ghosts. There are various ways of viewing them. In one both ϕ^{ab}_μ and ω^{ab}_μ can regarded as being on the same footing from the way they were originally introduced to localise the Gribov problem. However, from the equation of motion for ϕ^{ab}_μ, which was used to implement the horizon condition, ϕ^{ab}_μ is clearly not unrelated to the original gluon field itself and in some sense could be regarded as part of a more general spin-1 field such as

$$\bar{A}^a_\mu = A^a_\mu + \lambda\gamma^2 \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} \tag{7.1}$$

where λ is a constant, and which could be regarded as taking account of some of the more global aspects of the gauge fixing. Indeed the freezing of the effective coupling was dependent on the value of the ϕ^{ab}_μ 2-point function. So in some respects the infrared behaviour is guided by the inherent non-locality of a more general spin-1 field. Whilst this field $\bar{A}^a_\mu$ is clearly non-local it is similar to the second term in the expansion of the gauge invariant spin-1 field of [67] which is also non-local. In [67] the non-local structure resulting from the gauge invariant mass term in the Lagrangian was used to construct a vortex solution which underpinned the confinement process. Given that there is now numerical evidence for vortices present on the Gribov boundary (in the Coulomb gauge [68]), it might be possible to *construct* an explicit vortex solution in the Gribov-Zwanziger Lagrangian and study their dynamics in relation to the role they have in confinement. In contrast to the non-local mass term of [67], which is non-localizable since it would require an infinite number of extra fields, the non-locality of the Gribov formulation of non-abelian gauge fixing is localizable since only the finite set of fields $\{\phi^{ab}_\mu, \bar{\phi}^{ab}_\mu\}$ and $\{\omega^{ab}_\mu, \bar{\omega}^{ab}_\mu\}$ are required. A heuristic way of viewing this is to formally eliminate ϕ^{ab}_μ from (2.1), and ignoring all anticommuting ghost fields for the moment, then one has

$$L^{\rm GZ} = \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{C_A\gamma^4}{2} A^a_\mu \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} + \frac{dN_A\gamma^4}{2g^2} \tag{7.2}$$

which involves only one non-local term. In momentum space the quadratic part of (7.2) clearly leads to the Gribov gluon propagator. Returning to $\bar{A}^a_\mu$, one can now view the anticommuting Faddeev-Popov and Zwanziger ghosts from a different point of view to their original role, if one ignores their spins. In other words it seems that one can associate a ghost field with each term of $\bar{A}^a_\mu$ and, as we have shown here, each of these ghosts have similar infrared properties. For instance, both their propagators and form factors are equal at one loop and enhance at two loops in the $p^2 \to 0$ limit. Also the renormalization group invariant couplings defined from their interactions with the gluons both freeze at zero momentum to the same value. In this respect, aside from their differing spins they could be regarded as the first few terms of a ghost multiplet where the subsequent terms would be associated with higher order terms of $\bar{A}^a_\mu$ which would involve higher dimension non-local operators. Such operators might be the ones necessary to give the potential freezing in the effective coupling constants derived from the triple and quartic gluon vertices as suggested in [32, 33] or the restriction to the fundamental modular region.

Finally, we note that we have concentrated throughout in this article on studying the Gribov-Zwanziger Lagrangian in the Landau gauge only. However, it would be interesting to repeat the present analysis for other covariant gauges such as the other linear covariant Lorentz gauges and the maximal abelian gauge. Indeed for the former gauges there has been preliminary work in this direction, [69, 70], and it would be interesting to see, for instance, whether the power correction behaviour changed significantly and if the variation in magnitude of the residue of the $O(1/p^2)$ correction, if any, could be measured, say, on the lattice in a variety of gauges. Moreover, in the maximal abelian gauge it is possible to define an effective coupling constant from the Slavnov-Taylor structure of one of the gluon ghost vertices, [70]. Therefore, if one obtained the same freeze-out value for this effective coupling, it could be evidence for the abelian dominance hypothesis. We close by remarking that our study implies that the Gribov-Zwanziger Lagrangian has established infrared features which are generally not inconsistent with other methods. Specifically, these involve quantities such as $\alpha_S^{\mathrm{eff}}(p^2)$ which do not ultimately involve the running parameters $g(\mu)$ and $\gamma(\mu)$. Therefore, it would be interesting to apply (2.1) to other (infrared related) problems where such dependence on the running parameters cancels.

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A. Explicit 2-point functions.

In this appendix we record the explicit forms for the one loop corrections to each of the 2-point functions. Using the compact notation of (3.3) we have

$$
\begin{aligned}
X = - \Bigg[& -\frac{67\pi}{192}\sqrt{C_A}\gamma^2 - \frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{32}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right] \\
& + \Bigg[\frac{13\sqrt{2}}{32}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2} \ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
& \quad - \frac{13\sqrt{2}}{32}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2} \ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right] \\
& \quad + \frac{13\sqrt{2}}{16}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2} \ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \frac{13\sqrt{2}}{16}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right] \\
&\quad - \frac{25}{24}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\Bigg]\sqrt{C_A}\gamma^2 - \frac{39\pi}{64}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2} \\
&+ \left[\frac{77}{96}-\frac{53}{96}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]\right]\frac{C_A\gamma^4}{p^2}+\frac{37}{96}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right] \\
&- \frac{5C_A\gamma^4}{12(p^2)^2}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right] \\
&+ \Bigg[\frac{39\sqrt{2}}{128}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
&\quad - \frac{39\sqrt{2}}{128}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right] \\
&\quad + \frac{39\sqrt{2}}{64}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right] \\
&\quad - \frac{39\sqrt{2}}{64}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right] \\
&\quad + \frac{971}{288}+\frac{163}{192}\ln\left[\frac{(p^2)^2}{C_A\gamma^4}\right]+\frac{17}{96}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]-\frac{13}{6}\ln\left[\frac{p^2}{\mu^2}\right]\Bigg]p^2 \\
&+ \frac{13(p^2)^2}{384C_A\gamma^4}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]-\frac{21\pi(p^2)^2}{128\sqrt{C_A}\gamma^2} \\
&+ \Bigg[\frac{17\sqrt{2}}{192}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
&\quad - \frac{17\sqrt{2}}{192}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right] \\
&\quad + \frac{17\sqrt{2}}{96}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right] \\
&\quad + \frac{17\sqrt{2}}{96}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right] \\
&\quad + \frac{7}{96}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\Bigg]\frac{(p^2)^2}{\sqrt{C_A}\gamma^2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \left[\frac{5\sqrt{2}}{1536} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} + 1 \right]^{1/2} \ln \left[1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(p^2)^2}} \right] \right. \\
&\quad - \frac{5\sqrt{2}}{1536} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} + 1 \right]^{1/2} \ln \left[\frac{16 C_A \gamma^4}{(p^2)^2} \right] \\
&\quad + \frac{5\sqrt{2}}{768} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} + 1 \right]^{1/2} \ln \left[\left(1 + \sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} \right)^{1/2} - \sqrt{2} \right] \\
&\quad - \frac{5\sqrt{2}}{768} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{1/2} \tan^{-1} \left[\sqrt{2} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{-1/2} \right] \\
&\quad \left. - \frac{1}{96} \ln \left[1 + \frac{C_A \gamma^4}{(p^2)^2} \right] - \frac{1}{96} \ln \left[1 + \frac{(p^2)^2}{C_A \gamma^4} \right] \right] \frac{(p^2)^3}{C_A \gamma^4} \Bigg] C_A a \\
&- \left[\frac{4}{3} \ln \left(\frac{p^2}{\mu^2} \right) - \frac{20}{9} \right] p^2 T_F N_f a + O(a^2) \; . \qquad \text{(A.1)}
\end{aligned}
$$

For the mixed propagator we have $M = U$ with

$$
\begin{aligned}
U = &- \left[\left[\left[\frac{1}{64} \ln \left(1 + \frac{(p^2)^2}{C_A \gamma^4} \right) - \frac{31}{64} \right] - \frac{C_A \gamma^4}{96 (p^2)^2} \ln \left[1 + \frac{(p^2)^2}{C_A \gamma^4} \right] + \frac{179 \pi \sqrt{C_A} \gamma^2}{384 p^2} \right. \right. \\
&\quad - \frac{11 \sqrt{C_A} \gamma^2}{192 p^2} \tan^{-1} \left[\frac{\sqrt{C_A} \gamma^2}{p^2} \right] + \frac{7 \sqrt{4 C_A \gamma^4 - (p^2)^2}}{16 p^2} \tan^{-1} \left[- \frac{\sqrt{4 C_A \gamma^4 - (p^2)^2}}{p^2} \right] \\
&\quad - \frac{7 p^2 \sqrt{4 C_A \gamma^4 - (p^2)^2}}{64 C_A \gamma^4} \tan^{-1} \left[- \frac{\sqrt{4 C_A \gamma^4 - (p^2)^2}}{p^2} \right] - \frac{39 \pi p^2}{128 \sqrt{C_A} \gamma^2} - \frac{\pi (p^2)^3}{256 \sqrt{C_A^3} \gamma^6} \\
&\quad + \left[\frac{7\sqrt{2}}{64} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{1/2} \ln \left[1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(p^2)^2}} \right] \right. \\
&\qquad - \frac{7\sqrt{2}}{64} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{1/2} \ln \left[\frac{16 C_A \gamma^4}{(p^2)^2} \right] \\
&\qquad + \frac{7\sqrt{2}}{32} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{1/2} \ln \left[\left(1 + \sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} \right)^{1/2} - \sqrt{2} \right] \\
&\qquad + \frac{7\sqrt{2}}{32} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} + 1 \right]^{1/2} \tan^{-1} \left[\sqrt{2} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} - 1 \right]^{-1/2} \right] \\
&\qquad \left. \left. - \frac{1}{24} \tan^{-1} \left[\frac{\sqrt{C_A} \gamma^2}{p^2} \right] \right] \frac{p^2}{\sqrt{C_A} \gamma^2} \right. \\
&\quad + \left[- \frac{5\sqrt{2}}{256} \left[\sqrt{\left(1 + \frac{16 C_A \gamma^4}{(p^2)^2} \right)} + 1 \right]^{1/2} \ln \left[1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(p^2)^2}} \right] \right.
\end{aligned}
$$

$$+\frac{5\sqrt{2}}{256}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]$$

$$-\frac{5\sqrt{2}}{128}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]$$

$$+\frac{5\sqrt{2}}{128}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]$$

$$+\frac{5}{192}\ln\left(1+\frac{(p^2)^2}{C_A\gamma^4}\right)-\frac{1}{384}\ln\left(\frac{(p^2)^2}{C_A\gamma^4}\right)\bigg]\frac{(p^2)^2}{C_A\gamma^4}$$

$$+\left[\frac{\sqrt{2}}{512}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\right.$$

$$-\frac{\sqrt{2}}{512}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]$$

$$+\frac{\sqrt{2}}{256}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]$$

$$+\frac{\sqrt{2}}{256}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]$$

$$\left.+\frac{1}{64}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\right]\frac{(p^2)^3}{\sqrt{C_A^3\gamma^6}}\Bigg]C_A\gamma^2 a+O(a^2) \qquad \text{(A.2)}$$

The various colour channels of the ϕ^{ab}_μ 2-point function are

$$Q=-\left[-\frac{3\pi}{8}\sqrt{C_A}\gamma^2+\frac{3}{4}\sqrt{C_A}\gamma^2\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]-\frac{(p^2)^2}{4\sqrt{C_A}\gamma^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\right.$$
$$\left.+\frac{C_A\gamma^4}{8p^2}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]+\left[\frac{5}{4}-\frac{3}{8}\ln\left(\frac{[(p^2)^2+C_A\gamma^4]}{\mu^4}\right)\right]p^2\right]C_A a$$
$$+O(a^2) \qquad \text{(A.3)}$$

which is proportional to $\left(D_c(p^2)\right)^{-1}$

$$W=-\left[\frac{25\pi}{576}\sqrt{C_A}\gamma^2-\frac{17}{144}\sqrt{C_A}\gamma^2\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\right.$$
$$-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{72}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]$$
$$+\frac{C_A\gamma^4}{72(p^2)^2}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]$$

$$
\begin{aligned}
&+\frac{\pi\sqrt{C_A^3}\gamma^6}{144(p^2)^2}+\frac{1}{72}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]-\frac{19C_A\gamma^4}{576p^2}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]\\
&+\left[\frac{\sqrt{2}}{72}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\right.\\
&\quad-\frac{\sqrt{2}}{72}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]\\
&\quad+\frac{\sqrt{2}}{36}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]\\
&\quad-\frac{\sqrt{2}}{36}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]\\
&\quad\left.-\frac{1}{96}+\frac{7}{144}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]\right]p^2\\
&+\frac{(p^2)^2}{384C_A\gamma^4}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]-\frac{5\pi(p^2)^2}{1152\sqrt{C_A}\gamma^2}\\
&+\left[\frac{\sqrt{2}}{384}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\right.\\
&\quad-\frac{\sqrt{2}}{384}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]\\
&\quad+\frac{\sqrt{2}}{192}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]\\
&\quad+\frac{\sqrt{2}}{192}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]\\
&\quad\left.+\frac{5}{144}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\right]\frac{(p^2)^2}{\sqrt{C_A}\gamma^2}\\
&+\left[\frac{\sqrt{2}}{4608}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\right.\\
&\quad-\frac{\sqrt{2}}{4608}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]\\
&\quad+\frac{\sqrt{2}}{2304}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{\sqrt{2}}{2304}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right] \\
& - \frac{1}{1152}\ln\left[\frac{(p^2)^2}{C_A\gamma^4}\right] - \frac{1}{576}\ln\left[1+\frac{C_A\gamma^4}{(p^2)^2}\right]\Bigg]\Bigg]\frac{(p^2)^3}{C_A\gamma^4}\Bigg]\, a + O(a^2) \qquad \text{(A.4)}
\end{aligned}
$$

$$
\begin{aligned}
R = & -\Bigg[\frac{7\pi}{144}\sqrt{C_A}\gamma^2 - \frac{5}{36}\sqrt{C_A}\gamma^2\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right] \\
& - \frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{72}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right] \\
& + \frac{C_A\gamma^4}{18(p^2)^2}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right] - \frac{C_A\gamma^4}{32p^2} \\
& + \frac{25\pi\sqrt{C_A^3}\gamma^6}{576(p^2)^2} + \frac{7}{288}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right] - \frac{7C_A\gamma^4}{144p^2}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right] \\
& + \Bigg[\frac{\sqrt{2}}{72}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
& \quad - \frac{\sqrt{2}}{72}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right] \\
& \quad + \frac{\sqrt{2}}{36}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right] \\
& \quad - \frac{\sqrt{2}}{36}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right] \\
& \quad - \frac{1}{96} + \frac{11}{288}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]\Bigg]\Bigg]\, p^2 \\
& + \frac{\pi(p^2)^2}{288\sqrt{C_A}\gamma^2} + \frac{(p^2)^2}{288\sqrt{C_A}\gamma^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right] \\
& + \Bigg[\frac{\sqrt{2}}{1152}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right] \\
& \quad - \frac{\sqrt{2}}{1152}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right] \\
& \quad + \frac{\sqrt{2}}{576}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right] \\
& \quad - \frac{\sqrt{2}}{576}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]
\end{aligned}
$$

$$+\frac{1}{576}\ln\left[\frac{(p^2)^2}{C_A\gamma^4}\right]-\frac{1}{288}\ln\left[1+\frac{C_A\gamma^4}{(p^2)^2}\right]\Bigg]\frac{(p^2)^3}{C_A\gamma^4}\Bigg]\,a+O(a^2) \qquad \text{(A.5)}$$

and

$$\begin{aligned}
S = -\Bigg[&-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{12p^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]-\frac{17}{24}\sqrt{C_A}\gamma^2\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\\
&+\frac{25\pi}{96}\sqrt{C_A}\gamma^2+\frac{C_A\gamma^4}{12(p^2)^2}\sqrt{4C_A\gamma^4-(p^2)^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]\\
&+\frac{1}{12}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]+\frac{\pi}{24}\frac{\sqrt{C_A^3}\gamma^6}{(p^2)^2}-\frac{19C_A\gamma^4}{96p^2}\ln\left[1+\frac{(p^2)^2}{C_A\gamma^4}\right]\\
&+\Bigg[-\frac{1}{16}+\frac{7}{24}\ln\left[1+\frac{C_A\gamma^4}{(p^2)^2}\right]\\
&+\frac{\sqrt{2}}{12}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\\
&\quad-\frac{\sqrt{2}}{12}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]\\
&\quad+\frac{\sqrt{2}}{6}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]\\
&\quad-\frac{\sqrt{2}}{6}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]p^2\\
&+\Bigg[\frac{\sqrt{2}}{64}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(p^2)^2}}\right]\\
&\quad-\frac{\sqrt{2}}{64}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\frac{16C_A\gamma^4}{(p^2)^2}\right]\\
&\quad+\frac{\sqrt{2}}{32}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{1/2}\ln\left[\left(1+\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}\right)^{1/2}-\sqrt{2}\right]\\
&\quad+\frac{\sqrt{2}}{32}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}+1\right]^{1/2}\tan^{-1}\left[\sqrt{2}\left[\sqrt{\left(1+\frac{16C_A\gamma^4}{(p^2)^2}\right)}-1\right]^{-1/2}\right]\\
&\quad+\frac{5}{24}\tan^{-1}\left[\frac{\sqrt{C_A}\gamma^2}{p^2}\right]\Bigg]\frac{(p^2)^2}{\sqrt{C_A}\gamma^2}\\
&+\frac{(p^2)^2}{64C_A\gamma^4}\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{12p^2}\tan^{-1}\left[-\frac{\sqrt{4C_A\gamma^4-(p^2)^2}}{p^2}\right]-\frac{5(p^2)^2\pi}{192\sqrt{C_A}\gamma^2}
\end{aligned}$$

$$+ \left[\frac{\sqrt{2}}{768} \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} + 1 \right]^{1/2} \ln \left[1 + \sqrt{1 + \frac{16C_A\gamma^4}{(p^2)^2}} \right] \right.$$

$$- \frac{\sqrt{2}}{768} \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} + 1 \right]^{1/2} \ln \left[\frac{16C_A\gamma^4}{(p^2)^2} \right]$$

$$+ \frac{\sqrt{2}}{384} \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} + 1 \right]^{1/2} \ln \left[\left(1 + \sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} \right)^{1/2} - \sqrt{2} \right]$$

$$- \frac{\sqrt{2}}{384} \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1 \right]^{1/2} \tan^{-1} \left[\sqrt{2} \left[\sqrt{\left(1 + \frac{16C_A\gamma^4}{(p^2)^2}\right)} - 1 \right]^{-1/2} \right]$$

$$\left. + \frac{1}{192} \ln \left[\frac{C_A\gamma^4}{(p^2)^2} \right] - \frac{1}{96} \ln \left[1 + \frac{C_A\gamma^4}{(p^2)^2} \right] \right] \frac{(p^2)^3}{C_A\gamma^4} \right] \frac{a}{C_A} + O(a^2) \ . \qquad \text{(A.6)}$$

We note that in these expressions the explicit μ dependence appears only in a small set of terms, which can clearly be identified with the piece of the 2-point function deriving from the ultraviolet contribution.

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